



Math Virtual Learning

Precalculus with Trigonometry

April 27, 2020



Precalculus with Trigonometry

Lesson: April 27th, 2020

Objective/Learning Target:

Students will verify more difficult equations using Fundamental Trig Identities including Pythagorean, Quotient, and Reciprocal Identities.

Before you begin, make sure you remember the following identities from the previous lessons:

Let's Get Started:

Watch Video: Verifying Trigonometric Identities - [Verifying Trigonometric Identities](#)

Watch from the beginning for good review, but example #4 at 4:39 is the key example in this video.

Trigonometric Identities

Reciprocal Identities

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Trigonometric Identities

Please note that many of the identities can be written in more than one way.

Reciprocal Identities

$$\cot \theta = \frac{1}{\tan \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sin \theta = \frac{1}{\csc \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \cos \theta = \frac{1}{\sec \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta = 1 - \cos^2 \theta \quad \text{Pythagorean Identities}$$

$$\cos^2 \theta = 1 - \sin^2 \theta \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta = \sec^2 \theta - 1 \quad \tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta = \csc^2 \theta - 1 \quad 1 + \cot^2 \theta = \csc^2 \theta$$

Helpful Tips

1. Be on the lookout for any of the Pythagorean Identities
2. Converting to sine or cosine usually leads to other identities
3. Look at the format. Are there fractions on 1 side, but not the other? Are there more terms on 1 side? Use that as a clue as to what steps you may need to take.
4. Fractions, Fractions, Fractions. Remember how to divide fractions and that you need a common denominator to add fractions.
5. Use your Algebra “Tricks” such as factoring and conjugates

Trigonometric Identities

Reciprocal Identities $\cot \theta = \frac{1}{\tan \theta}$ $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$	Quotient Identities $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$
Pythagorean Identities $\sin^2 \theta + \cos^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$	

More Advanced Tips

Conjugate:

The conjugate of $1 - \sin x$ is $1 + \sin x$. When you multiply conjugates, you will get a difference of perfect squares. In this case $1 - \sin^2 x$.

Factoring (GCF):

Notice that the following expression has $\cos x$ in both terms. $\cos x + \cos x \cot^2 x$
You can factor that out and get $\cos x(1 + \cot^2 x)$

Factoring (Diff. of Sqs.):

Remember any term raised to an even power can be written as a perfect square. So the following expression is actually a difference of perfect squares. $\sin^4 x - \cos^4 x$
When factored you would get the following. $(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

Factoring (Trinomials):

The expression $\cos^2 x + 5\cos x - 6$ is a trinomial expression in quadratic format. It can be factored as $(\cos x + 6)(\cos x - 1)$.

Practice

Use the Fundamental Trigonometric Identities to verify the following equations.

$$1. \tan x + \frac{\cos x}{1 + \sin x} = \sec x$$

$$2. 1 - \frac{\sin^2 x}{1 + \cos x} = \cos x$$

$$3. \frac{\sin^2 x - \cos^2 x}{\sin x + \cos x} = \sin x - \cos x$$

$$4. \sin^4 x - \cos^4 x = 1 - 2\cos^2 x$$

$$5. \frac{\cos^2 x + 4\cos x + 4}{\cos x + 2} = \frac{2\sec x + 1}{\sec x}$$

Practice - ANSWERS

Use the Fundamental Trigonometric Identities to verify the following equations.

$$1. \tan x + \frac{\cos x}{1+\sin x} = \sec x$$

$$2. 1 - \frac{\sin^2 x}{1+\cos x} = \cos x$$

$$3. \frac{\sin^2 x - \cos^2 x}{\sin x + \cos x} = \sin x - \cos x$$

$$4. \sin^4 x - \cos^4 x = 1 - 2\cos^2 x$$

$$5. \frac{\cos^2 x + 4\cos x + 4}{\cos x + 2} = \frac{2\sec x + 1}{\sec x}$$

View the following video for worked out solutions and explanation for each of these problems. Skip to the times listed below for answers to specific problems.

Video: [Verifying Trigonometric Identities & Equations, Hard Examples With Fractions, Practice Problems](#)

1. 22:50

2. 25:40

3. 33:36

4. 45:00

5. 49:27

Additional Practice and Resources:

Additional Resource Videos:

[Verifying Trig Identities](#)

[Verifying a trigonometric Identities](#)

[Verifying Trigonometric Identities & Equations, Hard Examples With Fractions, Practice Problems](#) (There are many examples that weren't used in the practice problems)

Additional Practice:

[Trig Identities 1 \(with solutions\)](#)

[Trigonometric Identities \(Examples and Practice Problems\)](#)